## OCR

# Wednesday 18 June 2014 - Afternoon <br> <br> A2 GCE MATHEMATICS (MEI) 

 <br> <br> A2 GCE MATHEMATICS (MEI)}

4754/01A Applications of Advanced Mathematics (C4) Paper A

## QUESTION PAPER

Candidates answer on the Printed Answer Book.
OCR supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator


## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.


## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72 .
- The Printed Answer Book consists of $\mathbf{1 6}$ pages. The Question Paper consists of $\mathbf{8}$ pages. Any blank pages are indicated.
- This paper will be followed by Paper B: Comprehension.


## INSTRUCTIONTO EXAMS OFFICER/INVIGILATOR

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## Section A (36 marks)

1 Express $\frac{3 x}{(2-x)\left(4+x^{2}\right)}$ in partial fractions.
2 Find the first three terms in the binomial expansion of $(4+x)^{\frac{3}{2}}$. State the set of values of $x$ for which the expansion is valid.

3 Fig. 3 shows the curve $y=x^{3}+\sqrt{(\sin x)}$ for $0 \leqslant x \leqslant \frac{\pi}{4}$.


Fig. 3
(i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the $x$-axis and the line $x=\frac{\pi}{4}$, giving your answer to 3 decimal places.
(ii) Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say.
(i) Show that $\cos (\alpha+\beta)=\frac{1-\tan \alpha \tan \beta}{\sec \alpha \sec \beta}$.
(ii) Hence show that $\cos 2 \alpha=\frac{1-\tan ^{2} \alpha}{1+\tan ^{2} \alpha}$.
(iii) Hence or otherwise solve the equation $\frac{1-\tan ^{2} \theta}{1+\tan ^{2} \theta}=\frac{1}{2}$ for $0^{\circ} \leqslant \theta \leqslant 180^{\circ}$.

5 A curve has parametric equations $x=\mathrm{e}^{3 t}, y=t \mathrm{e}^{2 t}$.
(i) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of $t$. Hence find the exact gradient of the curve at the point with parameter $t=1$.
(ii) Find the cartesian equation of the curve in the form $y=a x^{b} \ln x$, where $a$ and $b$ are constants to be determined.

6 Fig. 6 shows the region enclosed by the curve $y=\left(1+2 x^{2}\right)^{\frac{1}{3}}$ and the line $y=2$.


Fig. 6
This region is rotated about the $y$-axis. Find the volume of revolution formed, giving your answer as a multiple of $\pi$.

Question 7 begins on page 4.

## Section B (36 marks)

7 Fig. 7 shows a tetrahedron ABCD . The coordinates of the vertices, with respect to axes Oxyz , are $\mathrm{A}(-3,0,0), \mathrm{B}(2,0,-2), \mathrm{C}(0,4,0)$ and $\mathrm{D}(0,4,5)$.


Fig. 7
(i) Find the lengths of the edges AB and AC , and the size of the angle CAB . Hence calculate the area of triangle ABC .
(ii) (A) Verify that $4 \mathbf{i}-3 \mathbf{j}+10 \mathbf{k}$ is normal to the plane ABC .
(B) Hence find the equation of this plane.
(iii) Write down a vector equation for the line through $D$ perpendicular to the plane $A B C$. Hence find the point of intersection of this line with the plane ABC.

The volume of a tetrahedron is $\frac{1}{3} \times$ area of base $\times$ height.
(iv) Find the volume of the tetrahedron ABCD .

8 Fig. 8.1 shows an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.


Fig. 8.1
The height of the water surface above the hole $t$ seconds after opening the hole is $h$ metres, where

$$
\frac{\mathrm{d} h}{\mathrm{~d} t}=-A \sqrt{h}
$$

and where $A$ is a positive constant. Initially the water surface is 1 metre above the hole.
(i) Verify that the solution to this differential equation is

$$
\begin{equation*}
h=\left(1-\frac{1}{2} A t\right)^{2} \tag{3}
\end{equation*}
$$

The water stops leaking when $h=0$. This occurs after 20 seconds.
(ii) Find the value of $A$, and the time when the height of the water surface above the hole is 0.5 m .

Fig. 8.2 shows a similar situation with a different barrel; $h$ is in metres.


Fig. 8.2
For this barrel,

$$
\frac{\mathrm{d} h}{\mathrm{~d} t}=-B \frac{\sqrt{h}}{(1+h)^{2}}
$$

where $B$ is a positive constant. When $t=0, h=1$.
(iii) Solve this differential equation, and hence show that

$$
\begin{equation*}
h^{\frac{1}{2}}\left(30+20 h+6 h^{2}\right)=56-15 B t . \tag{7}
\end{equation*}
$$

(iv) Given that $h=0$ when $t=20$, find $B$.

Find also the time when the height of the water surface above the hole is 0.5 m .

## END OF QUESTION PAPER

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# Wednesday 18 June 2014 - Afternoon <br> <br> A2 GCE MATHEMATICS (MEI) 

 <br> <br> A2 GCE MATHEMATICS (MEI)}

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

## QUESTION PAPER

Candidates answer on the Question Paper.
OCR supplied materials:
Duration: Up to 1 hour

- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
- Rough paper


| Candidate <br> forename | Candidate <br> surname |  |
| :--- | :--- | :--- | :--- |


| Centre number |  |  |  |  |  | Candidate number |  |  |  |  |
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## INSTRUCTIONS TO CANDIDATES

- The Insert will be found inside this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do not write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are not required to hand in these notes with your question paper.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 18.
- This document consists of 8 pages. Any blank pages are indicated.

1 The example illustrated in Table 1 in the article referred to the members of a racing club.
This question is about a similar club and the equivalent table is given below. The club has 150 members and each of them belongs to one of the groups $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ or S .

For each group the probability of an accident and the average cost is given in the table below.

| Group | P | Q | R | S |
| :--- | :---: | :---: | :---: | :---: |
| Proportion of people | $10 \%$ | $20 \%$ | $40 \%$ | $30 \%$ |
| Probability of an accident | 0.1 | 0.15 | 0.05 | 0.2 |
| Average cost per accident | $£ 5000$ | $£ 2000$ | $£ 1000$ | $£ 500$ |

(i) Complete the table below.
(ii) Calculate the premium if each member of the club pays the same amount and the total of the premiums is equal to the total average cost of accidents per year.
(iii) The club management decides that the total of the premiums should be $50 \%$ more than the total average cost of accidents per year, and that each group should pay an amount proportional to its contribution to the total average cost.

Calculate the premium for a member of group $S$.

| 1 (i) | Group $\mathbf{P}$ <br> Number of people $\mathbf{Q}$ <br> Average number of accidents in a year  <br> $\mathbf{R}$ $\mathbf{S}$ <br> Average cost of accidents per year $£ 7500$ |
| :--- | :--- | :---: | :---: | :---: | :---: |



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2 In lines 97 to 100, the article says
"Most insurance companies have a maximum no-claims discount of $65 \%$. One way of interpreting this practice is that the figure arrived at by applying the maximum no-claims discount is actually the basic cost of the insurance, and that drivers who have not earned the so-called discount are actually paying a surcharge."

A new driver without any no-claims discount pays $k$ times "the basic cost of the insurance".
Find the value of $k$.


## PLEASE DO NOT WRITE IN THIS SPACE

3 On June 1st 2007 Louise paid her first car insurance premium. She did not have any no-claims discount.
She retained her policy with the same insurance company. She had no accidents and so did not make any claim on her insurance. Her no-claims discount followed the pattern in Table 7. Apart from her no-claims discount, her basic premium remained the same.

On May 31st 2014, Louise calculated that so far her no-claims discount had saved her $£ 3875$.
What premium did Louise pay on June 1st 2007 ?


4 In lines 68 to 69, the article says
"Clearly the role of inexperience goes down with age. A possible mathematical model is that it decays exponentially."

This question investigates this in the case of male drivers. The relevant part of Table 4 is reproduced here.

| Age (in years) | $\mathbf{1 8 . 0}$ | $\mathbf{2 0 . 5}$ | $\mathbf{2 3 . 0}$ | $\mathbf{2 7 . 5}$ | $\mathbf{4 5 . 0}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Male \% | 36 | 15 | 6 | 2.5 | 1 |

A model is proposed in which

$$
y=a+b \mathrm{e}^{-k(x-17)},
$$

where

- $x$ is the age of the driver,
- $y$ is the percentage of accidents in which inexperience played a role,
- $\quad a, b$ and $k$ are constants to be determined.
(i) Draw a sketch graph of $y$ against $x$ for values of $x$ between 18 and 45 .
(ii) Explain why the graph justifies putting $a=1$.
(iii) Taking $a=1$ and using the first two data points, it is estimated that $b=50.5$ and $k=0.3665$. Investigate whether these values are consistent with the point $(23,6)$.


| 4 (ii) |  |
| :---: | :---: |
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| 4 (iii) |  |
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5 In this question the number of points on a driver's licence for motoring offences is denoted by $n$.
When calculating a driver's premium, a particular insurance company takes account of such offences by multiplying the premium by an amount $M$, where

$$
\begin{aligned}
& M=1 \quad \text { if } n \leqslant 3 \\
& M=2^{\frac{n}{6}} \quad \text { if } 3<n<12 .
\end{aligned}
$$

A driver who is insured with this company is paying a premium of $£ 520$. Initially he has no points on his licence.

He is convicted of a speeding offence and so receives 3 points on his licence. He is then convicted for dangerous driving and receives a further 6 points on his licence.

## Calculate his premium

(A) after his first offence,
(B) after his second offence.


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